RADIATION COOLING OF A DENSE, HIGH-β PLASMA

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In the currently projected quasistationary thermonuclear systems with plasma densities of $n \sim 10^{17}$-$10^{18}$ cm$^{-3}$, the separation of the plasma from the walls by means of a magnetic field is a technically complicated problem, since it requires the establishment of megagauss fields. One possibility is to confine the plasma by means of the vessel walls and to use the magnetic field only to suppress the transverse thermal conductivity [1]; this can be achieved with fairly moderate magnetic fields (such that $\beta = 16\pi nT/H^2 \gg 1$). In addition to the obvious features related to the direct contact between the plasma and the walls, this nonmagnetic confinement produces significantly new properties in the bulk behavior of the plasma. Let us suppose that the plasma is in a long cylindrical tube of radius R. Since the interesting confinement times are much longer than the inertia time $R/c_s$ ($c_s$ is the velocity of sound), the gas-kinetic pressure $p = 2nT$ in a plasma with $\beta \gg 1$ must at all times be uniform over the cross section of the tube. Thus, the plasma density near the relatively cold walls is much greater than that in the center of the system. This can lead to an important increase in the role of the bremsstrahlung emitted by the plasma, since the volume intensity of radiation $Q_r$, which is proportional to $n^2T^{1/2}$, increases as $n^{3/2}$ when $T \sim n^{-1}$. The reduction in the energy lifetime of the plasma thus produced can mean that nonmagnetic confinement is unusable in principle, since for a thermonuclear plasma with a temperature $T \sim 10^4$ eV the radiation cooling time $\tau_r = 3nT/Q_r$ even for a D–T mixture is only 30 times greater than the required confinement time given by the Lawson criterion.

Suppose that at the initial instant of time we have at the center a hot plasma with a temperature $T_0$ and density $n_0$ and a magnetic field $H_0$ directed along the tube. We assume that the plasma temperature near the wall is equal to zero. In the center, the hot plasma is strongly magnetized so that the quantity $\delta_0 = \omega_{Hi}\tau_i \gg 1$ ($\omega_{Hi}$ is the ion cyclotron frequency and $\tau_i^{-1}$ is the ion collision frequency), but the magnetic pressure is small: $\beta_0 = 16\pi n_0 T_0/H_0^2 \gg 1$. The problem consists in finding the time $\tau_E$ in which the plasma loses a significant part of its initial energy $W_0 \sim n_0 T_0 R^2$ (we consider only the transverse energy losses so that all quantities refer to unit length of the system). The thermal losses come from radiation and transverse thermal conductivity in the plasma. Since the pressure 2nT in a high-β plasma always remains uniform, the plasma must become redistributed over the cross section as it cools, i.e., there is a radial plasma flow. The time changes in the plasma parameters are described by the equations [2]

$$\frac{\partial}{\partial r}(nT) = 0, \quad \frac{dn}{dt} + \frac{n}{r}\frac{\partial}{\partial r}(rv) = 0; \tag{1}$$

$$\frac{d}{dt}\left(\frac{H}{n}\right) = \frac{c^2}{4\pi r}\frac{\partial}{\partial r}\left(\frac{r}{\sigma_\perp}\frac{\partial H}{\partial r}\right) + \frac{c}{er}\frac{\partial}{\partial r}\left(r\frac{\beta_\wedge^{uT}}{n}\frac{\partial T}{\partial r}\right); \tag{2}$$

$$3n\frac{dT}{dt} - 2T\frac{dn}{dt} = \frac{1}{r}\frac{\partial}{\partial r}r\left(\varkappa_\perp\frac{\partial T}{\partial r} + \frac{c\beta_\wedge^{uT}}{4\pi ne}\frac{\partial H}{\partial r}\right) - Q_r, \tag{3}$$

where v is the radial flow velocity of the plasma, $\kappa_\perp$ and $\sigma_\perp$ are the thermal and electrical conductivities of the plasma across the magnetic field, and $\beta_\wedge^{uT}$ is a coefficient which defines the component of the thermal force perpendicular to **∇T** and **H**. The volume intensity of the bremsstrahlung from the plasma is

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$$Q_r = bn^2T^{1/2} \approx 10^{-13}n^2T^{1/2},$$

where Q is in $eV/cm^3 \cdot sec$, n is in $cm^{-3}$, and T is in eV. In the thermal balance equation (3) we have omitted unimportant terms connected with ohmic and viscous dissipation.

Since the full solution of the system (1)-(3) can only be obtained by numerical integration, we shall give a qualitative analysis of the various plasma cooling conditions. For very small systems (R tending to zero) the radiation from the plasma can, of course, be neglected. The energy lifetime of the plasma is then on the order of $R^2/\chi_0$, where $\chi_0 \sim cT_0/eH_0\delta_0$ is the thermal diffusivity of a magnetized hot plasma [2]. For large R, on the other hand, $\tau_E$ is bounded from above by the radiation cooling time of a hot plasma $\tau_0 = 3n_0T_0/bn_0^2T_0^{1/2}$.

There is a certain region of radius values where the energy losses from the system are governed by the radiation from the thin layer of cold plasma near the walls (the thickness is much less than R) and the cooling time $\tau_E$ in this case is much less than either $\tau_0$ or the diffusion time $R^2/\chi_0$. The cooling of the hot plasma occurs as a result of the convective flow of heat from the center toward the walls, i.e., of the adiabatic expansion of the hot plasma. The plasma flow involved can be described as follows. The wall layer of low-temperature plasma cools rapidly as a result of radiation and it contracts (since the plasma pressure must be uniform). Thermal conductivity leads to an extraction of heat from the next layer of plasma, which then cools, and so the process continues. Since the thickness of the dense cooled plasma is small in comparison to the radius of the tube, the hot plasma is, as it were, "eaten" by the walls.

The expansion rate of the hot plasma can be found by solving the following model problem. Suppose that the plasma parameters depend on a single coordinate x (when the thickness of the transition layer between the hot and cool plasmas is small compared to R, the motion of the plasma can be assumed to be uniform). At $x = -\infty$ we have hot plasma, and at $x = +\infty$ we have cold plasma. If we neglect the radiation from the hot plasma (which is justifiable when $\tau_E \ll \tau_0$), we see that a stationary "cooling wave" caused by the radiation from the transition layer propagates through the plasma; the convective flow of heat from the hot plasma must balance the total radiated energy. In order to find the profile of this wave it is convenient to go over to dimensionless variables in (1)-(3). We therefore take $n_0$, $T_0$, and $H_0$ as our units of density, temperature, and magnetic field, respectively. We measure length in units of $(\chi_0, \tau_0)^{1/2}$ and velocity in units of $(\chi_0/\tau_0)^{1/2}$. For a stationary flow we get from (1) that

$$T = n^{-1},\ nv = v_0 \tag{4}$$

($v_0$ is the velocity of the hot plasma). In a high-β plasma, the frozen-in nature of the magnetic field is mainly destroyed by the action of the thermal force [the second term on the right side of (2)]. But in our case we can neglect this force and assume that the magnetic field is frozen into the plasma:

$$H = n \tag{5}$$

[the large factor $(M/m)^{1/2}$ in the thermal diffusivity $\chi_0$ of a magnetized hot plasma is the parameter]. Under condition (5), the magnetization parameter $\delta = \omega_{Hi}\tau_i$ depends only on the temperature and is equal to $\delta_0 T^{3/2}$. For $T > \delta_0^{-2/3}$, the plasma is magnetized and in dimensionless variables (3) can be written as

$$5v_0\frac{dT}{dx} = \frac{d}{dx}\left(T^{-1/2}\frac{dT}{dx}\right) - T^{-3/2} \tag{6a}$$

[here, we have used (4) and (5) and omitted the small correction to the heat flow produced by the thermal force]. When $\delta < 1$, i.e., $T < \delta_0^{-2/3}$, the magnetic field does not affect the thermal conductivity of the plasma* and in place of (6a) we get

$$5v_0\frac{dT}{dx} = \delta_0^2\frac{d}{dx}\left(T^{5/2}\frac{dT}{dx}\right) - T^{-3/2}. \tag{6b}$$

*For simplicity, we do not consider the narrow temperature region corresponding to the transition from ion to electron thermal conductivity.

Integrating (6a) and (6b) over the coordinate and neglecting the diffusion flow of heat into the cold plasma (for which the condition is $\tau_E \ll R^2/\chi_0$), we find that

$$v_0 = \frac{1}{5} \int_{-\infty}^{+\infty} T^{-3/2} dx \tag{7}$$

[the radiation from the hot and cold plasma is "cut off" and only the transition region contributes to the integral (7) .

In order to calculate $v_0$ we consider separately the high-temperature region (which we call region 1), where the radiation is unimportant and the convective and diffusion heat flows are balanced, and the low-temperature region (region 2), where we can neglect the convective heat flow. We estimate the order of magnitude of dT/dx as $T/l$ and we have from (6a) that in region 1

$$v_0 T/l \sim T^{1/2}/l^2, \; l \sim 1/v_0 T^{1/2}. \tag{8}$$

Comparing now the terms on the right side of (6a), we find the boundary $T_1$ between regions 1 and 2: $T_1 \sim v_0^{-2/3}$ [as we shall see later, $T_1 \gg \delta_0^{-2/3}$, so that the whole of region 1 is described by (6a)]. In region 2, it follows from (6a) that for $T > \delta_0^{-2/3}$,

$$T^{1/2}/l^2 \sim T^{-3/2}, \; l \sim T, \tag{9}$$

and from (6b) for $T < \delta_0^{-2/3}$,

$$\delta_0^2 T^{7/2}/l^2 \sim T^{-3/2}, \quad l \sim \delta_0 T^{5/2}. \tag{10}$$

By means of (8)-(10), we can now evaluate the integral (7) which defines $v_0$. The main contribution comes from the temperature range $T \sim \delta_0^{-2/3}$, where $\omega_{Hi}\tau_i \sim 1$,

$$v_0 \sim \int T^{-3/2} dx \sim \int dT\, l(T)\, T^{-5/2} \sim \delta_0^{1/3}. \tag{11}$$

In these calculations we have completely neglected the magnetic pressure in comparison with the gas-kinetic pressure of the plasma. But however large the value of $\beta_0$, the magnetic pressure must increase as the temperature goes down if the field is frozen into the plasma, and for sufficiently small T it will become greater than the plasma pressure. From the condition that the total pressure should be uniform we can estimate that the condition $n = T^{-1}$ will be satisfied up to temperatures $T \sim \beta_0^{-1/2}$. At lower temperatures n and H will no longer vary. It can thus be seen that (11) is valid if $\beta_0^{1/2} \gtrsim \delta_0^{2/3}$. In the opposite case, which usually holds for plasma parameters of practical interest (where, for example, $n_0 \sim 10^{18}$ cm$^{-3}$, $T_0 \sim 10$ keV, $H_0 \sim 10^5$ G, $\beta_0 \sim 10^2$, $\delta_0 \sim 10^3$), the main contribution to the radiation comes from the temperature range $T \sim \beta_0^{-1/2}$. The expansion rate of the hot plasma is then $v_0 \sim \beta_0^{1/4}$.

Going back to dimensional quantities, we now get the following estimate for the energy lifetime of the hot plasma:

$$\tau_E \sim \beta_0^{-1/4} R\, (\tau_0/\chi_0)^{1/2}. \tag{12}$$

It is interesting to note that the cooling time of a hot plasma proves to be proportional to the radius of the system R. These cooling conditions only occur, of course, if the magnitude of $\tau_E$ estimated from (12) is smaller than both $R^2/\chi_0$ and $\tau_0$. This condition leads to the following range for the transverse dimensions*:

$$\beta_0^{-1/4} (\chi_0 \tau_0)^{1/2} \leqslant R \leqslant \beta_0^{1/4} (\chi_0 \tau_0)^{1/2}.$$

The shape of the $\tau_E$ versus R curves for the different types of cooling are shown in Fig. 1. If we apply these results to a plasma with thermonuclear parameters and assume that

*We note that the stationary cooling wave which models the plasma behavior over the given range of parameters depends to an important extent on the boundary conditions and these have to be determined from the actual data. An incorrect choice of these boundary conditions led to the erroneous result in [3].

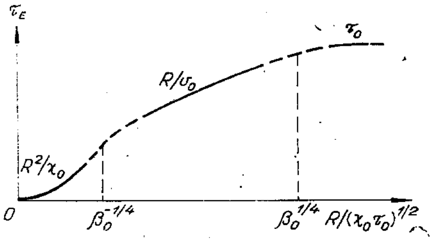

Fig. 1

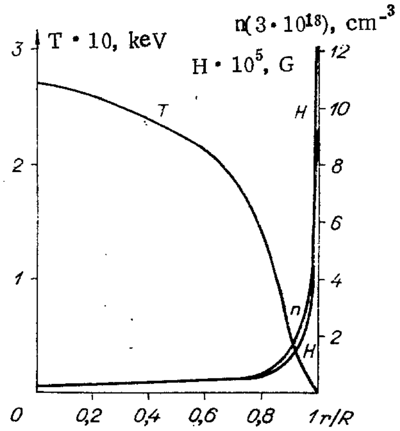

Fig. 2

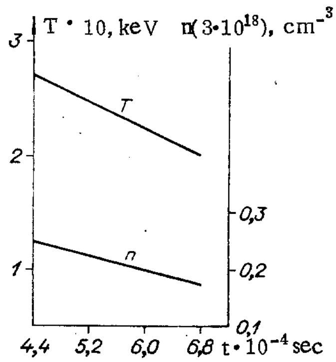

Fig. 3

the required energy confinement time is of the order of $\tau_0/30$, we find that for $\beta_0 < 30^2 \approx 10^3$ the radiation is not important in the plasma energy balance. Thus, the quantity $\tau_E$ remains of the order of $R^2/\chi_0$, as was found earlier from numerical calculations of the parameters of a thermonuclear reactor with nonmagnetic confinement [4, 5]. For $\beta_0 \gtrsim 10^3$, this increase in the total radiation significantly reduces the plasma cooling time. As an example, we now give the results obtained by computer integration of (1)-(3) for this case.

The actual problem is as follows. At the initial instant of time a uniform plasma with a density $n = 3 \cdot 10^{18}$ cm$^{-3}$ is in a tube with a radius of R = 17 cm. The magnetic field is also uniform and equal to $10^5$ G. An external heat source with a characteristic duration of $\Delta t = 6 \cdot 10^{-5}$ sec is then switched on. The heating is accompanied by the expulsion of the plasma and the magnetic field from the center toward the walls [we might note that it is more convenient in the numerical integration to use the equation of motion of the plasma rather than the first equation in (1)]. At the time $t_1 = 4.4 \cdot 10^{-4}$ sec, when the source can be considered to be already switched off, the radial distributions of the temperature, density, and magnetic field are as shown in Fig. 2. The plasma parameters in the center are as follows: $n_0 = 7.5 \cdot 10^{17}$ cm$^{-3}$, $H_0 = 2.5 \cdot 10^4$ G, and $T_0$ = 27 keV, so that $\beta_0 = 2.7 \cdot 10^3$. The magnetic pressure is only important in a thin layer right next to the wall, where $\beta \ll 1$. The characteristic time associated with the thermal conductivity is $R^2/\chi_0 \approx 2 \cdot 10^{-3}$ sec, and the radiation time is $\tau_0 \approx 6.4 \cdot 10^{-3}$ sec. Thus, we can expect from our estimates that (12) will be valid for the plasma cooling time $\tau_E$ in this case.

Figure 3 shows how the plasma temperature and density change with time after $t = t_1$. It can be seen that the cooling is almost adiabatic. The average volume radiation intensity over the cross section is about 12 times greater than the intensity at the center and the diffusion heat flow to the wall is $^1/_3$ of the total radiation from the plasma. By the time $t_2 = 6.8 \cdot 10^{-4}$ sec, the plasma energy is one-half of its initial value (at $t = t_1$) so that the energy lifetime $\tau_E = 2.4 \cdot 10^{-4}$ sec [the estimate from (12) gives $\beta_0^{-1/4} R(\tau_0/\chi_0)^{1/2} \approx 5 \cdot 10^{-4}$ sec].

The special features of the radiation from a high-β plasma also appear in the stationary problem, where the heat losses from the plasma are balanced by the external heat source. Suppose that at the initial moment of time we have a cold plasma which uniformly fills the entire cross section of the tube. The plasma density is $\bar{n}$ and the magnetic field is $\bar{H}$. We switch on a constant external heat source with a characteristic volume intensity $Q_H$. We have to decide whether a stationary state can be set up in a high-β plasma for a sufficiently large value of $Q_H$. We suppose that the magnetic field remains frozen into the plasma while the steady state is being set up. It then follows from our estimates that when $R > \beta_0^{-1/4} (\chi_0\tau_0)^{1/2}$ the thermal conductivity of the hot plasma which occupies the major part of the volume and acquires almost all the energy of the external source cannot balance the radiation from the cold layer near the walls, so that a stationary state proves to be impossible. We shall find the conditions that this imposes on the initial parameters $\bar{n}$, $\bar{H}$, and R. The quantities $\chi_0$, $\tau_0$, and $\beta_0$ depend on the plasma density and temperature at the center ($n_0$ and $T_0$), and in the stationary state these are determined from the conservation of the number of particles and from the energy balance. The condition for energy balance can, of course, be written here as

$$Q_H R^2/\chi_0 \sim n_0 T_0.$$

In order to find the density profile in the stationary state we have to solve an equation similar to (6a) except that the convective heat flow is replaced by $Q_H$. Simple estimates similar to those given above show that $n_0 \sim \beta_0^{-1/4}\bar{n}$ (and $H_0 \sim \beta_0^{-1/4}\bar{H}$). If we now take the thermal diffusivity of the hot plasma to be $\chi_0 \sim Mc^2 a n_0/e^2 H_0^2 T_0^{1/2}$ (where we have explicitly introduced the ion collision frequency $\nu_i = an/T^{3/2}$), we find that the condition for the existence of a stationary solution is independent of the heat intensity $Q_H$ and the initial plasma density $\bar{n}$ and has the form

$$\bar{H}R < (Mc^2 a/e^2 b)^{1/2} \approx 10^5 \ \text{G}\cdot\text{cm}.$$

The hot-plasma parameters in the stationary state are

$$T_0 \sim Q_H^2 \bar{H}^4 R^4/\bar{n}^4 (Mc^2 a/e^2), \ H_0 \sim n_0 \bar{H}/\bar{n},$$
$$n_0 \sim \bar{n}^2 (Mc^2 a/e^2)^{2/3}/Q_H^{2/3} \bar{H}^{2/3} R^{4/3}.$$

We note, in conclusion, that these features of the radiation cooling of a high-β plasma are related to the way that the classical Coulomb thermal conductivity of a plasma depends on the magnetic field, the temperature, and the density. These features do not occur, for example, in the case of a Bohm thermal conductivity, where $\chi \sim cT/eH$.

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